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TECHNICAL REPORT

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CONFIDENCED NORMAL AND LOGNORMAL RELIABILITY FOR ANY SAMPLE SIZE

BY

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CONFIDENCED NORMAL AND LOGNORMAL RELIABILITY FOR ANY SAMPLE SIZE

ABSTRACT

For a given mission life (x), an exact 100C% lower confidence limit on population reliability is found.

The conditions and/or assumptions underlying this analysis are: (1) The population is either normal or lognormal, (2) A sample of two or more representative failures is available. Neyman's general classical method of confidence intervals is used. This is possible because the distribution of the reliability estimator is a one parameter distribution which is independent of both failure density parameters and mission life.

Cross-Reference Data

Reliability
Confidence Interval
Neyman
Normal
Lognormal
Estimator
Parameters
Exact Sampling Distribution
Independence
Jacobian

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1. INTRODUCTION

Important aspects of this analysis are:

a. Two-parameter distributions are dealt with.

b. Both parameters are estimated from the sample (i.e., no parameter is "assumed to be known)".

c. A representative sample of size 2 or greater is acceptable data.

The results of the analysis will be presented first for the sake of brevity.

The derivations will be carried out for the normal population only, since the lognormal case is only trivially different.

The general procedure in the derivation of equation (1) will be:

1. Find the joint density of $\hat{\mu}$ and $\hat{\sigma}$.

2. Obtain the density and distribution function of $\hat{R}$ by making a change of variable.

3. Use Neyman's method to obtain a one-sided confidence interval for R.

4. Simplify the resulting expressions.

2. NOTATION

n = sample size

μ, σ = normal population mean and standard deviation

$\hat{\mu}, \hat{\sigma}$ = estimators of μ and σ

x = mission life

R = Population reliability

$\hat{R}$ = estimator of R,

$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}t^2} dt$ = standard normal distribution function

z_α = standard normal fractile function (inverse of distribution function)

μ^*, σ^* = estimates of μ and σ

R^* = estimate of R

C = confidence level

μ_ℓ, σ_ℓ = paramaters of the lognormal distribution

$\mu_\ell^*, \sigma_\ell^*$ = estimates of μ_ℓ, σ_ℓ (mean and standard deviation of the logs of the data)

R_c = confidenced reliability

3. RESULTS

a. Normal population:

The equation which yields confidenced reliability R_c is:

$$\frac{(1-c)\Gamma\left(\frac{n-1}{2}\right)}{2\left(\frac{n}{2}\right)^{\frac{n-1}{2}}} =$$

$$\int_0^\infty s^{n-2} e^{-\frac{ns^2}{2}} \Phi\left[\sqrt{n}\left(s\left(\frac{x-\mu^*}{\sigma^*}\right)-z_{1-R_c}\right)\right] ds \qquad (1)$$

Equation (1) is solved for z_{1-R_c} and then —

$$R_c = 1 - \Phi\left(z_{1-R_c}\right)$$

The inverse problem may just as easily be solved, i.e., if one stipulates desired confidenced reliability, one can solve equation (1) for the necessary mission life x.

b. Lognormal population:

The equation which yields confidenced reliability R_c is: $\frac{(1-c)\Gamma\left(\frac{n-1}{2}\right)}{2\left(\frac{n}{2}\right)^{\frac{n-1}{2}}} =$

$$\int_0^\infty s^{n-2} e^{-\frac{ns^2}{2}} \Phi\left[\sqrt{n}\left(s\left(\frac{\ln x-\mu_\ell^*}{\sigma_\ell^*}\right)-z_{1-R_c}\right)\right] ds \qquad (2)$$

The foregoing remarks apply equally to equation (2).

In order to implement the numerical solution of these equations, the following programs are needed:

1. A program to evaluate the standard normal distribution function.
2. An integrating program (such as Romberg integration).
3. A univariate nonlinear equation solver (a reliable method being the bisection or midpoint method).

Needless to say, these equations are of very little value if one does not have a digital computer available.

4. DERIVATION OF RESULTS

Let x_i $(i = 1, 2, \cdots n)$ be normally distributed with mean μ and standard derivation σ.

If x is the desired mission life, population reliability is given by:

$$R = 1 - \Phi\left(\frac{x - \mu}{\sigma}\right) \tag{3}$$

and the reliability estimator is given by:

$$\hat{R} = 1 - \Phi\left(\frac{x - \hat{\mu}}{\hat{\sigma}}\right) \tag{4}$$

Equation (4) gives $\hat{R}$ as a function of $\hat{\mu}$ and $\hat{\sigma}$; therefore the distribution of $\hat{R}$ may be obtained if one first finds the joint distribution of $\hat{\mu}$ and $\hat{\sigma}$.

Form a bivariate change of variable using the dummy variable u.

$$\hat{R} = 1 - \Phi\left(\frac{x - \hat{\mu}}{\hat{\sigma}}\right) \tag{4}$$

$$u = \hat{\sigma} \tag{5}$$

Finding the inverse of this relation:

$$\Phi\left(\frac{x-\hat{\mu}}{\hat{\sigma}}\right) = 1-\hat{R}$$

$$\frac{x-\hat{\mu}}{\hat{\sigma}} = z_{1-\hat{R}}$$

$$\hat{\mu} = x - \hat{\sigma}\, z_{1-\hat{R}} \tag{6}$$

the inverse relation is:

$$\hat{\mu} = x - u\, z_{1-\hat{R}} \tag{7}$$

$$\hat{\sigma} = u \tag{8}$$

now,

$$h(\hat{R}, u) = f(\hat{\mu}, \hat{\sigma}) \left| \frac{\partial(\hat{\mu}, \hat{\sigma})}{\partial(\hat{R}, u)} \right| \tag{9}$$

Where h and f are joint density functions.

The Jacobian is obtained first, using equations (7) and (8):

$$J = \frac{\partial(\hat{\mu}, \hat{\sigma})}{\partial(\hat{R}, u)} = \begin{vmatrix} \frac{\partial \hat{\mu}}{\partial \hat{R}} & \frac{\partial \hat{\mu}}{\partial u} \\ \frac{\partial \hat{\sigma}}{\partial \hat{R}} & \frac{\partial \hat{\sigma}}{\partial u} \end{vmatrix}$$

$$= \begin{vmatrix} \frac{\partial \hat{\mu}}{\partial \hat{R}} & \frac{\partial \hat{\mu}}{\partial u} \\ 0 & 1 \end{vmatrix} = \frac{\partial \hat{\mu}}{\partial \hat{R}}$$

$$= -u \frac{d}{d\hat{R}} z_{1-\hat{R}} \tag{10}$$

but $\Phi(z_\alpha) = \alpha$ (11)

$$\therefore \frac{d\alpha}{dv} = \Phi'(z_\alpha) \frac{dz_\alpha}{dv}$$

$$\therefore \frac{dz_\alpha}{dv} = \frac{1}{\Phi'(z_\alpha)} \frac{d\alpha}{dv}$$

but

$$\Phi(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{x} e^{-\frac{1}{2}t^2} dt$$

$$\therefore \Phi'(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}x^2}$$

$$\frac{d\mathfrak{z}_\alpha}{dv} = \frac{1}{\frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2}\mathfrak{z}_\alpha^2}} \frac{d\alpha}{dv}$$

$$= \sqrt{2\pi}\, e^{\frac{1}{2}\mathfrak{z}_\alpha^2} \frac{d\alpha}{dv} \qquad (12)$$

Using equations (10) and (12)

$$J = -u \cdot \sqrt{2\pi}\, e^{\frac{1}{2}\mathfrak{z}_{1-\hat{R}}^2} \cdot (-1) \qquad (13)$$

$$= \sqrt{2\pi}\, u\, e^{\frac{1}{2}\mathfrak{z}_{1-\hat{R}}^2}$$

Therefore from equations (9) and (13):

$$h(\hat{R}, u) = f(\hat{\mu}, \hat{\sigma}) \sqrt{2\pi}\, u\, e^{\frac{1}{2}\mathfrak{z}_{1-\hat{R}}^2}$$

or

$$h(\hat{R},\hat{\sigma}) = f(\hat{\mu},\hat{\sigma})\sqrt{2\pi}\,\hat{\sigma}\,e^{\frac{1}{2}z^2_{1-\hat{R}}} \tag{14}$$

After $f(\hat{\mu},\hat{\sigma})$ is determined, $\hat{\sigma}$ may be integrated out (in equation (14)) to obtain the density of $\hat{R}$.

Now, x_i is normal with mean μ and standard deviation σ .

$\therefore$ $y = \dfrac{n\hat{\sigma}^2}{\sigma^2}$ is chi-square distributed with $n-1$ degrees of freedom:

$$\therefore g(y) = \frac{1}{\Gamma\left(\frac{n-1}{2}\right)2^{\frac{n-1}{2}}}\, y^{\frac{n-1}{2}-1}\, e^{-\frac{y}{2}}$$

The density of $\hat{\sigma}$ is therefore given by:

$$g(\hat{\sigma}) = g(y)\left|\frac{dy}{d\hat{\sigma}}\right|$$

but $y = \dfrac{n\hat{\sigma}^2}{\sigma}$

$$\therefore \frac{dy}{d\hat{\sigma}} = \frac{2n\hat{\sigma}}{\sigma^2}$$

$$\therefore g(\hat{\sigma}) = \frac{1}{\Gamma\left(\frac{n-1}{2}\right) 2^{\frac{n-1}{2}}} \left(\frac{n\hat{\sigma}^2}{\sigma^2}\right)^{\frac{n-3}{2}} e^{-\frac{1}{2}\frac{n\hat{\sigma}^2}{\sigma^2}} \cdot \frac{2n\hat{\sigma}}{\sigma^2}$$

$\therefore$ The density of $\hat{\sigma}$ is given by:

$$g(\hat{\sigma}) = \frac{2\left(\frac{n}{2}\right)^{\frac{n-1}{2}}}{\sigma\Gamma\left(\frac{n-1}{2}\right)} \left(\frac{\hat{\sigma}}{\sigma}\right)^{n-2} e^{-\frac{n}{2}\left(\frac{\hat{\sigma}}{\sigma}\right)^2} \tag{15}$$

Now, $\hat{\mu}$ is normally distributed with mean μ and standard deviation $\frac{\sigma}{\sqrt{n}}$; therefore, the density of $\hat{\mu}$ is given by:

$$r(\hat{\mu}) = \frac{1}{\sqrt{2\pi}\,\frac{\sigma}{\sqrt{n}}} e^{-\frac{1}{2}\left(\frac{\hat{\mu}-\mu}{\frac{\sigma}{\sqrt{n}}}\right)^2}$$

$$= \frac{1}{\sigma}\sqrt{\frac{n}{2\pi}}\, e^{-\frac{n}{2}\left(\frac{\hat{\mu}-\mu}{\sigma}\right)^2} \tag{16}$$

Now, since $\hat{\mu}$ and $\hat{\sigma}$ are independent random variables, equations (15) and (16) yield the joint density of

$\hat{\mu}$ and $\hat{\sigma}$:

$$f(\hat{\mu},\hat{\sigma}) = g(\hat{\sigma}) \times r(\hat{\mu})$$

$$= \frac{2}{\sigma} \frac{\left(\frac{n}{2}\right)^{\frac{n-1}{2}}}{\Gamma\left(\frac{n-1}{2}\right)} \left(\frac{\hat{\sigma}}{\sigma}\right)^{n-2} e^{-\frac{n}{2}\left(\frac{\hat{\sigma}}{\sigma}\right)^2} \times$$

$$\frac{1}{\sigma}\sqrt{\frac{n}{2\pi}}\, e^{-\frac{n}{2}\left(\frac{\hat{\mu}-\mu}{\sigma}\right)^2}$$

$$\therefore f(\hat{\mu},\hat{\sigma}) = \frac{2}{\sigma^2}\sqrt{\frac{n}{2\pi}} \frac{\left(\frac{n}{2}\right)^{\frac{n-1}{2}}}{\Gamma\left(\frac{n-1}{2}\right)} \left(\frac{\hat{\sigma}}{\sigma}\right)^{n-2} e^{-\frac{n}{2}\left[\left(\frac{\hat{\sigma}}{\sigma}\right)^2 + \left(\frac{\hat{\mu}-\mu}{\sigma}\right)^2\right]} \qquad (17)$$

Substituting this expression into equation (14):

$$h(\hat{R},\hat{\sigma}) = \frac{2}{\sigma^2}\sqrt{\frac{n}{2\pi}} \frac{\left(\frac{n}{2}\right)^{\frac{n-1}{2}}}{\Gamma\left(\frac{n-1}{2}\right)} \left(\frac{\hat{\sigma}}{\sigma}\right)^{n-2} e^{-\frac{n}{2}\left[\left(\frac{\hat{\sigma}}{\sigma}\right)^2 + \left(\frac{\hat{\mu}-\mu}{\sigma}\right)^2\right]} \times$$

$$\sqrt{2\pi}\, \hat{\sigma}\, e^{\frac{1}{2} z^2_{1-\hat{R}}}$$

$$h(\hat{R},\hat{\sigma}) = \frac{\frac{2\sqrt{n}}{\sigma}\left(\frac{n}{2}\right)^{\frac{n-1}{2}}}{\Gamma\left(\frac{n-1}{2}\right)}\left(\frac{\hat{\sigma}}{\sigma}\right)^{n-1} e^{-\frac{n}{2}\left[\left(\frac{\hat{\sigma}}{\sigma}\right)^2+\left(\frac{\hat{\mu}-\mu}{\sigma}\right)^2\right]+\frac{1}{2}z^2_{1-\hat{R}}} \tag{18}$$

but $\hat{\mu} = x - \hat{\sigma} z_{1-\hat{R}}$ Equation (6)

and $\mu = x - \sigma z_{1-R}$

$$\therefore \quad \frac{\hat{\mu}-\mu}{\sigma} = \frac{\sigma z_{1-R} - \hat{\sigma} z_{1-\hat{R}}}{\sigma}$$

$$= z_{1-R} - \frac{\hat{\sigma}}{\sigma} z_{1-\hat{R}} \tag{19}$$

let $$K = \frac{2\sqrt{n}\left(\frac{n}{2}\right)^{\frac{n-1}{2}}}{\Gamma\left(\frac{n-1}{2}\right)} \tag{20}$$

Therefore, using equations (18), (19), and (20):

$$h(\hat{R},\hat{\sigma}) = \frac{K}{\sigma}\left(\frac{\hat{\sigma}}{\sigma}\right)^{n-1} e^{-\frac{n}{2}\left[\left(\frac{\hat{\sigma}}{\sigma}\right)^2+\left(z_{1-R}-\frac{\hat{\sigma}}{\sigma}z_{1-\hat{R}}\right)^2\right]+\frac{1}{2}z^2_{1-\hat{R}}} \tag{21}$$

Now, $\hat{\sigma}$ is integrated out to obtain the density of $\hat{R}$:

$$h(\hat{R}) = \int_0^\infty \frac{K}{\sigma}\left(\frac{\hat{\sigma}}{\sigma}\right)^{n-1} e^{-\frac{n}{2}\left[\left(\frac{\hat{\sigma}}{\sigma}\right)^2 + \left(z_{1-R} - \frac{\hat{\sigma}}{\sigma} z_{1-\hat{R}}\right)^2\right] + \frac{1}{2} z^2_{1-\hat{R}}} \, d\hat{\sigma} \tag{22}$$

let $s = \dfrac{\hat{\sigma}}{\sigma}$

$\therefore \; d\hat{\sigma} = \sigma \, ds$

$\therefore$ The density of $\hat{R}$ is given by:

$$h(\hat{R}) = K \int_0^\infty s^{n-1} e^{-\frac{n}{2}\left[s^2 + \left(z_{1-R} - s z_{1-\hat{R}}\right)^2\right] + \frac{1}{2} z^2_{1-\hat{R}}} \, ds \tag{23}$$

Since s is a dummy variable of integration and $\hat{R}$ is the argument of h, the only numbers upon which the form of h is dependent are n and R. Therefore, the density of the reliability estimator is a one parameter (R) density which is independent of the failure density population parameters and mission life.

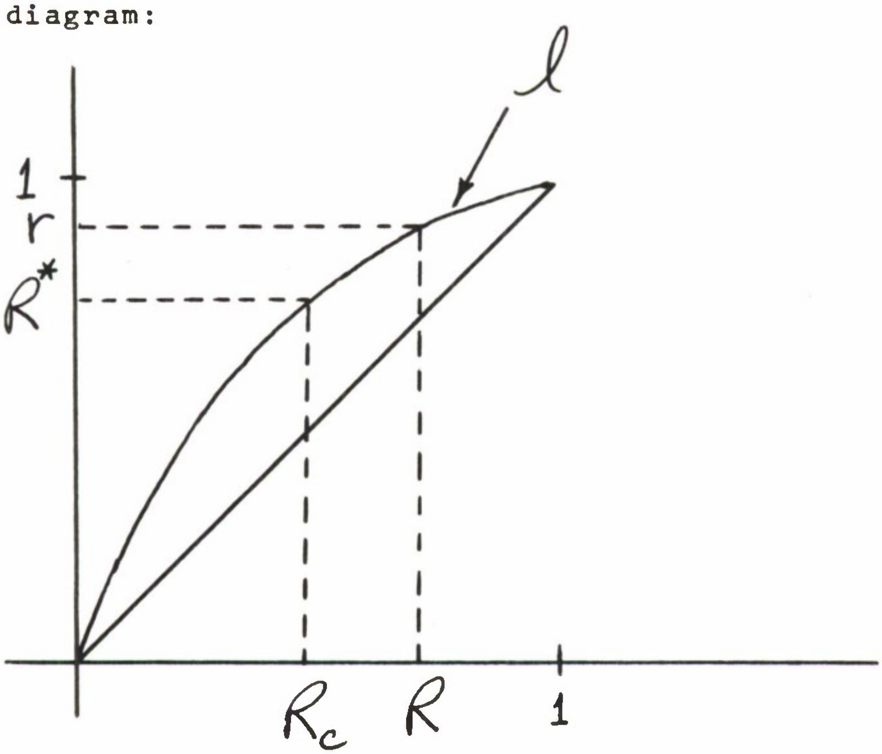
Changing the argument of h to avoid confusion and adding the subscript R to h to indicate its dependence on the population reliability R, the density of $\hat{R}$ is:

$$h_R(v) = K \int_0^\infty s^{n-1} e^{-\frac{n}{2}\left[s^2 + (s z_{1-v} - z_{1-R})^2\right] + \frac{1}{2} z_{1-v}^2} \, ds \tag{24}$$

The distribution function of $\hat{R}$ is therefore given by:

$$H_R(r) = K \int_0^r \int_0^\infty s^{n-1} e^{-\frac{n}{2}\left[s^2 + (s z_{1-v} - z_{1-R})^2\right] + \frac{1}{2} z_{1-v}^2} \, ds \, dv \tag{25}$$

The Neyman method of finding a one-sided confidence interval for R may be explained through the following diagram:



The curve ℓ is determined by:

$$P(\hat{R} < r \mid R) = C$$

or

$$H_R(r) = C \tag{26}$$

The information in the failure data is introduced at this point:

$$R^* = 1 - \Phi\left(\frac{x - \mu^*}{\sigma^*}\right) \tag{27}$$

R_c is then determined by solving equation (28)

$$H_{R_c}(R^*) = C \tag{28}$$

In order to solve equation (28), the distribution function of $\hat{R}$ given by equation (25) should be simplified.

$$H_R(r) = K \int_0^r \int_0^\infty s^{n-1} e^{-\frac{n}{2}\left[s^2 + (s z_{1-v} - z_{1-R})^2\right] + \frac{1}{2} z_{1-v}^2} \, ds \, dv \tag{25}$$

Changing the order of integration:

$$H_R(r) = K \int_0^{\infty} \int_0^{r} s^{n-1} e^{-\frac{n}{2}\left[s^2 + \left(s z_{1-v} - z_{1-R}\right)^2\right] + \frac{1}{2} z_{1-v}^2} \, dv \, ds$$

$$= K \int_0^{\infty} s^{n-1} e^{-\frac{n}{2} s^2} \int_0^{r} e^{-\frac{n}{2}\left(s z_{1-v} - z_{1-R}\right)^2 + \frac{1}{2} z_{1-v}^2} \, dv \, ds \qquad (29)$$

But from equation (12)

$$dz_{\alpha} = \sqrt{2\pi}\, e^{\frac{1}{2} z_{\alpha}^2} \, d\alpha$$

$$\therefore \quad dz_{1-v} = -\sqrt{2\pi}\, e^{\frac{1}{2} z_{1-v}^2} \, dv \qquad (30)$$

Substituting (30) into (29):

$$H_R(r) = \frac{-K}{\sqrt{2\pi}} \int_0^{\infty} s^{n-1} e^{-\frac{n s^2}{2}} \int_0^{r} e^{-\frac{1}{2}\left(\sqrt{n}\, s z_{1-v} - \sqrt{n}\, z_{1-R}\right)^2} \, dz_{1-v} \, ds \qquad (31)$$

Now let

$$u = \sqrt{n}\, s z_{1-v} \qquad (32)$$

$$\therefore \quad dz_{1-v} = \frac{du}{\sqrt{n}\, s} \qquad (33)$$

Substituting (32) and (33) into (31)

$$H_R(r) = \frac{-K}{\sqrt{2\pi n}} \int_0^\infty s^{n-2} e^{-\frac{ns^2}{2}} \int_{\infty}^{\sqrt{n}\, s z_{1-r}} e^{-\frac{1}{2}\left(u - \sqrt{n}\, z_{1-R}\right)^2} du\, ds$$

$$H_R(r) = \frac{K}{\sqrt{n}} \int_0^\infty s^{n-2} e^{-\frac{ns^2}{2}} \cdot \frac{1}{\sqrt{2\pi}} \int_{\sqrt{n}\, s z_{1-r}}^{\infty} e^{-\frac{1}{2}\left(u - \sqrt{n}\, z_{1-R}\right)^2} du\, ds$$

$$H_R(r) = \frac{K}{\sqrt{n}} \int_0^\infty s^{n-2} e^{-\frac{ns^2}{2}} \cdot \frac{1}{\sqrt{2\pi}} \int_{\sqrt{n}\left(s z_{1-r} - z_{1-R}\right)}^{\infty} e^{-\frac{1}{2}u^2} du\, ds$$

$$H_R(r) = \frac{K}{\sqrt{n}} \int_0^\infty s^{n-2} e^{-\frac{ns^2}{2}} \left\{1 - \Phi\left[\sqrt{n}\left(s z_{1-r} - z_{1-R}\right)\right]\right\} ds \qquad (34)$$

Separating (34) into two integrals:

$$H_R(r) = \frac{K}{\sqrt{m}} \int_0^\infty s^{m-2} e^{-\frac{ms^2}{2}} ds - \frac{K}{\sqrt{m}} \int_0^\infty s^{m-2} e^{-\frac{ms^2}{2}} \Phi\left[\sqrt{m}\left(s z_{1-r} - z_{1-R}\right)\right] ds \tag{35}$$

Let $$I = \int_0^\infty s^{m-2} e^{-\frac{ms^2}{2}} ds \tag{36}$$

To evaluate I, make the change of variable: $u = \frac{ms^2}{2}$

$$\therefore du = ms\,ds, \quad s^2 = \frac{2u}{m}, \quad s = \left(\frac{2u}{m}\right)^{1/2}$$

$$s^{m-2} = \left(\frac{2u}{m}\right)^{\frac{m-2}{2}}$$

$$\therefore I = \int_0^\infty \left(\frac{2u}{m}\right)^{\frac{m-2}{2}} e^{-u} \frac{du}{m\left(\frac{2u}{m}\right)^{1/2}}$$

$$= \int_0^\infty \frac{1}{m}\left(\frac{2u}{m}\right)^{\frac{m-3}{2}} e^{-u} du$$

$$= \frac{1}{m}\left(\frac{2}{m}\right)^{\frac{m-3}{2}} \int_0^\infty u^{\frac{m-1}{2}-1} e^{-u} du$$

$$= \frac{1}{m}\left(\frac{2}{m}\right)^{\frac{m-3}{2}} \Gamma\left(\frac{m-1}{2}\right) \tag{37}$$

but from (20), $K = \dfrac{2\sqrt{m}\left(\frac{m}{2}\right)^{\frac{m-1}{2}}}{\Gamma\left(\frac{m-1}{2}\right)}$

$$\therefore \frac{KI}{\sqrt{m}} = \frac{2\left(\frac{m}{2}\right)^{\frac{m-1}{2}}}{\Gamma\left(\frac{m-1}{2}\right)} \cdot \frac{1}{m}\left(\frac{2}{m}\right)^{\frac{m-3}{2}}\Gamma\left(\frac{m-1}{2}\right) = 1 \tag{38}$$

using (38) in (35)

$$H_R(r) = 1 - \frac{2\left(\frac{m}{2}\right)^{\frac{m-1}{2}}}{\Gamma\left(\frac{m-1}{2}\right)}\int_0^\infty s^{m-2} e^{-\frac{ms^2}{2}} \Phi\left[\sqrt{m}\left(s z_{1-r} - z_{1-R}\right)\right] ds \tag{39}$$

Using equation (39), equation (28) now becomes:

$$C = 1 - \frac{2\left(\frac{m}{2}\right)^{\frac{m-1}{2}}}{\Gamma\left(\frac{m-1}{2}\right)}\int_0^\infty s^{m-2} e^{-\frac{ms^2}{2}} \Phi\left[\sqrt{m}\left(s z_{1-R^*} - z_{1-R_c}\right)\right] ds$$

or

$$\frac{(1-C)\Gamma\left(\frac{m-1}{2}\right)}{2\left(\frac{m}{2}\right)^{\frac{m-1}{2}}} = \int_0^\infty s^{m-2} e^{-\frac{ms^2}{2}} \Phi\left[\sqrt{m}\left(s z_{1-R^*} - z_{1-R_c}\right)\right] ds \tag{40}$$

But from equation (27), $z_{1-R^*} = \dfrac{x-\mu^*}{\sigma^*}$

Therefore equation (1) is obtained:

$$\frac{(1-C)\Gamma\left(\frac{n-1}{2}\right)}{2\left(\frac{n}{2}\right)^{\frac{n-1}{2}}} = \int_0^\infty s^{n-2} e^{-\frac{ns^2}{2}} \Phi\left[\sqrt{n}\left(s\left(\frac{x-\mu^*}{\sigma^*}\right) - z_{1-R_C}\right)\right] ds$$

Equation (2) is obtained analogously; one simply thinks of working entirely within the domain of the logarithms of the data points.

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13. ABSTRACT

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14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
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For a given mission life (x), an exact 100C% lower confidence limit on population reliability is found. The conditions and/or assumptions underlying this analysis are: (1) The population is either normal or lognormal, (2) A sample of two or more representative failures is available. Neyman's general classical method of confidence intervals is used. This is possible because the distribution of the reliability estimator is a one parameter distribution which is independent of both failure density parameters and mission life.

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Neyman
Normal
Lognormal
Estimator
Parameters
Exact Sampling Distribution
Independence
Jacobian

Distribution Unlimited

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